Fundamental limits for Quantum Thermodynamics

1111.3834 Horodecki, Oppenheim
1111.3882 Brandao, Horodecki, Oppenheim, Renes, Spekkens

Resource Theories

- Water
- Entanglement Bennett et. al.
- Purity Horodecki^2, Oppenheim (2003)
- Asymmetry Gour, Spekkens (2008)
- Discord / Deficit Horodecki$^{\otimes 3}$, Sen$^{\otimes 2}$, J.O. ....
(2002/2004)

Resource Theories

Theory	Entanglement	Purity	Asymmetry
Class of Operations $\mathcal{C}$	LOCC	Noisy $\frac{\mathbb{1}}{d}$, U	$T(g) \circ \mathcal{E} \circ T^\dagger(g) = \mathcal{E}$
Useless states S	seperable σ	$\mathbb{1}/d$	$\sigma = T(g)\sigma T^\dagger(g)$
Monotones	$\inf_\sigma R(\rho\Vert\sigma)$	$R(\rho\Vert\frac{\mathbb{I}}{d}) = N - S(\rho)$	$\inf_\sigma R(\rho\Vert\sigma)$

"Discord" Thermodynamics

$$S(\rho\|\sigma) = -\mathrm{tr}\rho\log\sigma + \mathrm{tr}\rho\log\rho$$

$$M(\rho) := \inf_{\sigma\in S} S(\rho\|\sigma)$$

Generic measure under the class C
(monotone)

Unique measure if C reversible

- Horodecki, Oppenheim
quant-ph/0207177

Eg. Discord as a resource theory

Horodecki$^{\otimes 3}$ JO, Sens$^{\otimes 2}$, Synak   quant-ph/0207168

Class of Operations

$\mathcal{C}^{all}$: $\quad \mathcal{U}$, Dephasing in basis

$\mathcal{C}^{0}$, $\overrightarrow{\mathcal{C}}$, $\overleftrightarrow{\mathcal{C}}$ $\quad \mathcal{U}_A, \mathcal{U}_B$, dephasing + 0, 1, 2 way communication

Useless States

$S^{all} := \mathbb{1}/d$

S^{0}: "classical" states

S^{1}: c-q states,

S^{2}: IBP-states

Horodecki$^{\otimes 3}$, JO, Sen$^{\otimes 2}$, Synak quant ph/0410090

$$M(\rho) := \inf_{\sigma \in S} S(\rho \| \sigma)$$

S^{all}	$\mathbb{1}/d$	$N - S$
S^{0}	"classical" states	Δ^{0} also Modi et al and Piani et al
S^{1}	c-q states	$\bar{\Delta}$
S^{2}	IPP states	Δ ?
S^{sep}	sep	$E_R(\rho)$, Brandao | Plenio
S^{g}	g	frameness (Gour, Spekkens)
$S^{thermal}$	gibbs	$F = E - TS$

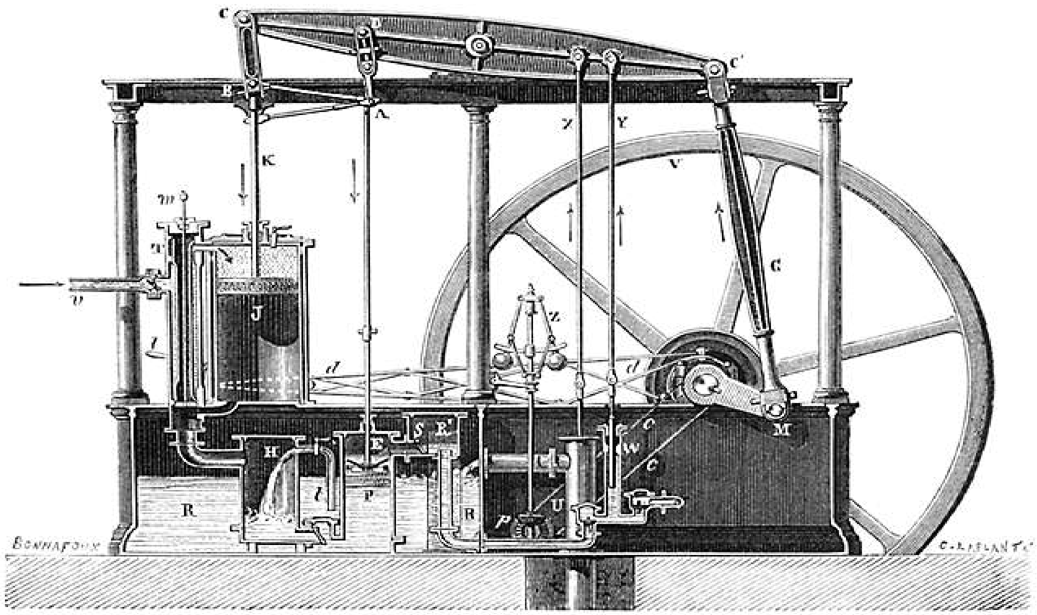
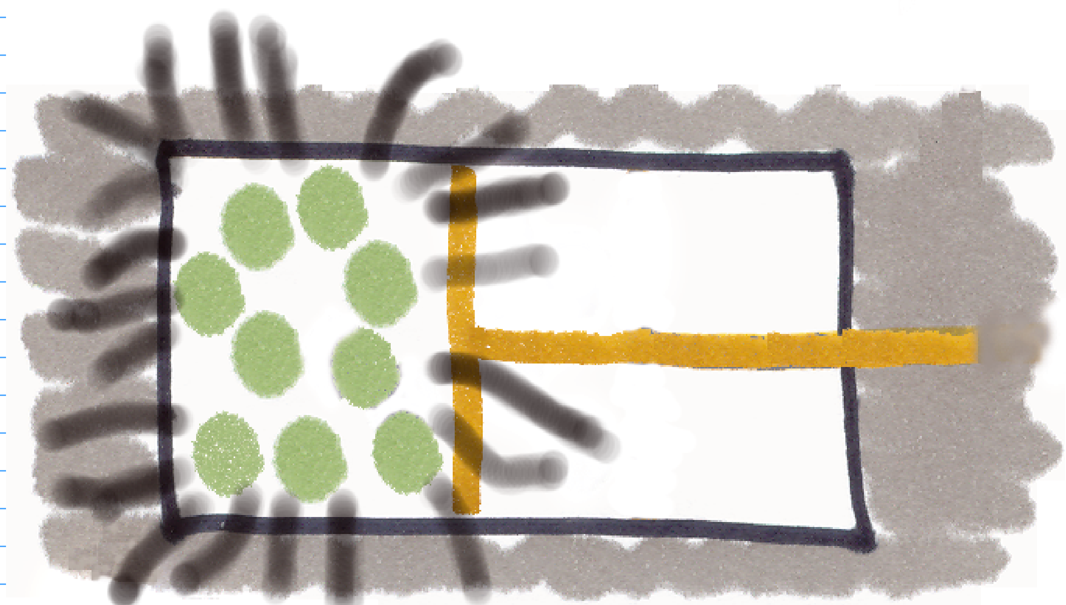


Fig. 59. — Machine à balancier de Watt.

v. Tuyau de prise de vapeur; T, tiroir; J, cylindre; H, condenseur; PE pompe d'épuisement; WY pompe alimentaire de la chaudière UX pompe d'alimentation de la bâche R; p Z régulateur; dd excentrique; ABCD parallélogramme; GM bielle et manivelle; V volant.

1st Wave

Carnot (1824)
Joule (1843)
Kelvin (1849)
Clausius (1854)



2nd Wave
(stat mech)

Maxwell (1871)
Boltzman (1875)
Gibbs (1876)

Its called thermodynamics because we take the thermodynamic limit!

System size $\longrightarrow \infty$

number of particles

Thermodynamics in the opposite extreme

Finite size (micro, nano)

and/or quantum

Outline

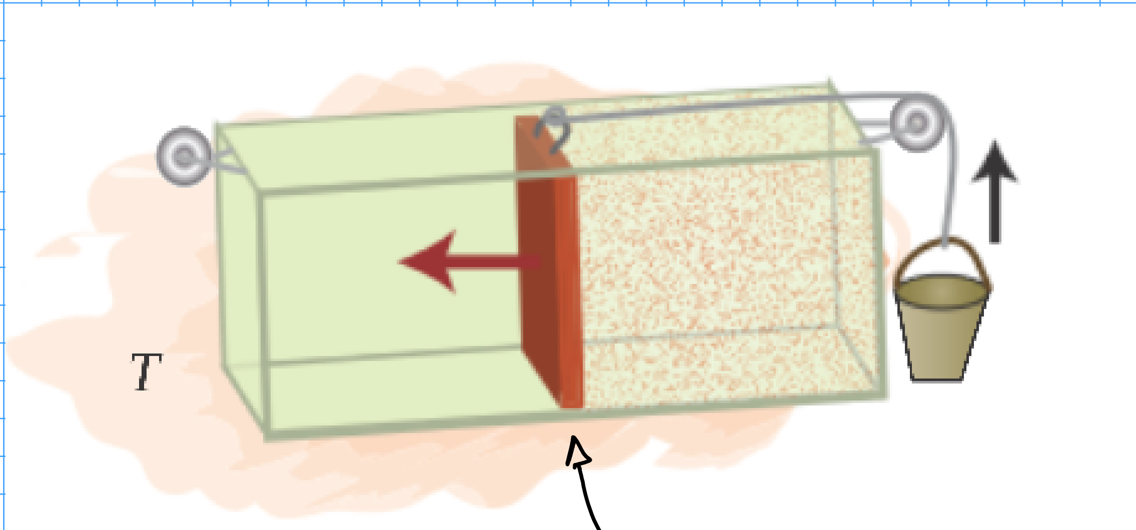
Result: two free energies

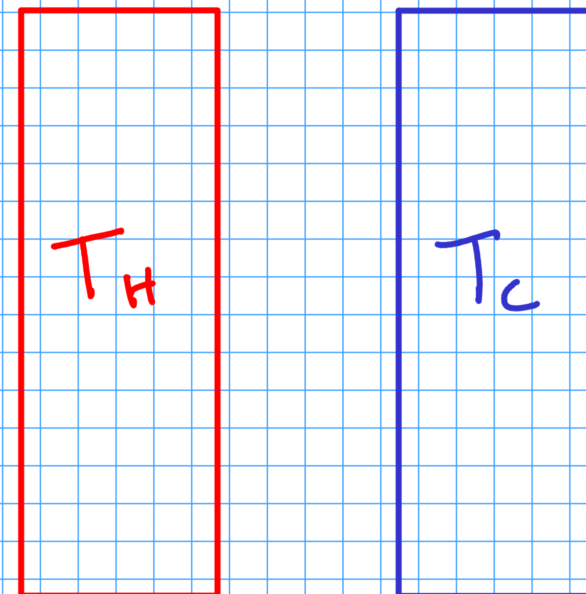
Fundamental laws of quantum Thermodynamics

Paradigm: Resource theories
Single-shot information theory

Sketch ideas: What thermodynamical transitions are possible?
When reversible?
Smallest efficient engines

Thermodynamics





out of equilibrium

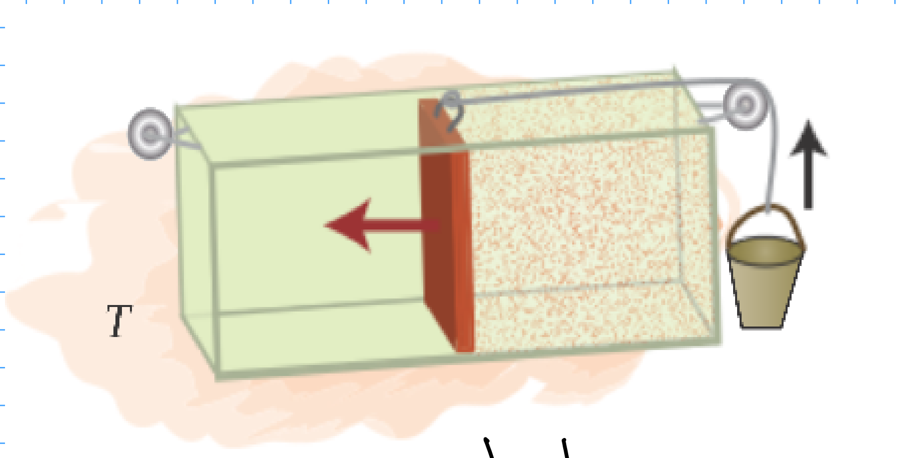
$$F = E - TS$$

$$W = F(\rho_{initial}) - F(\rho_{final})$$

$\rho_{initial} \to \rho_{final}$ only if $\Delta F \geq 0$

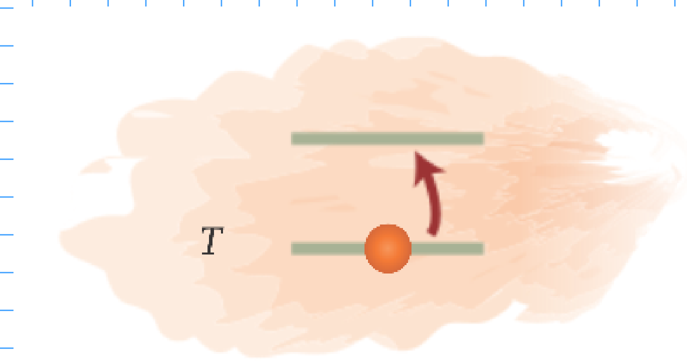
Work

Macro



weight

Micro



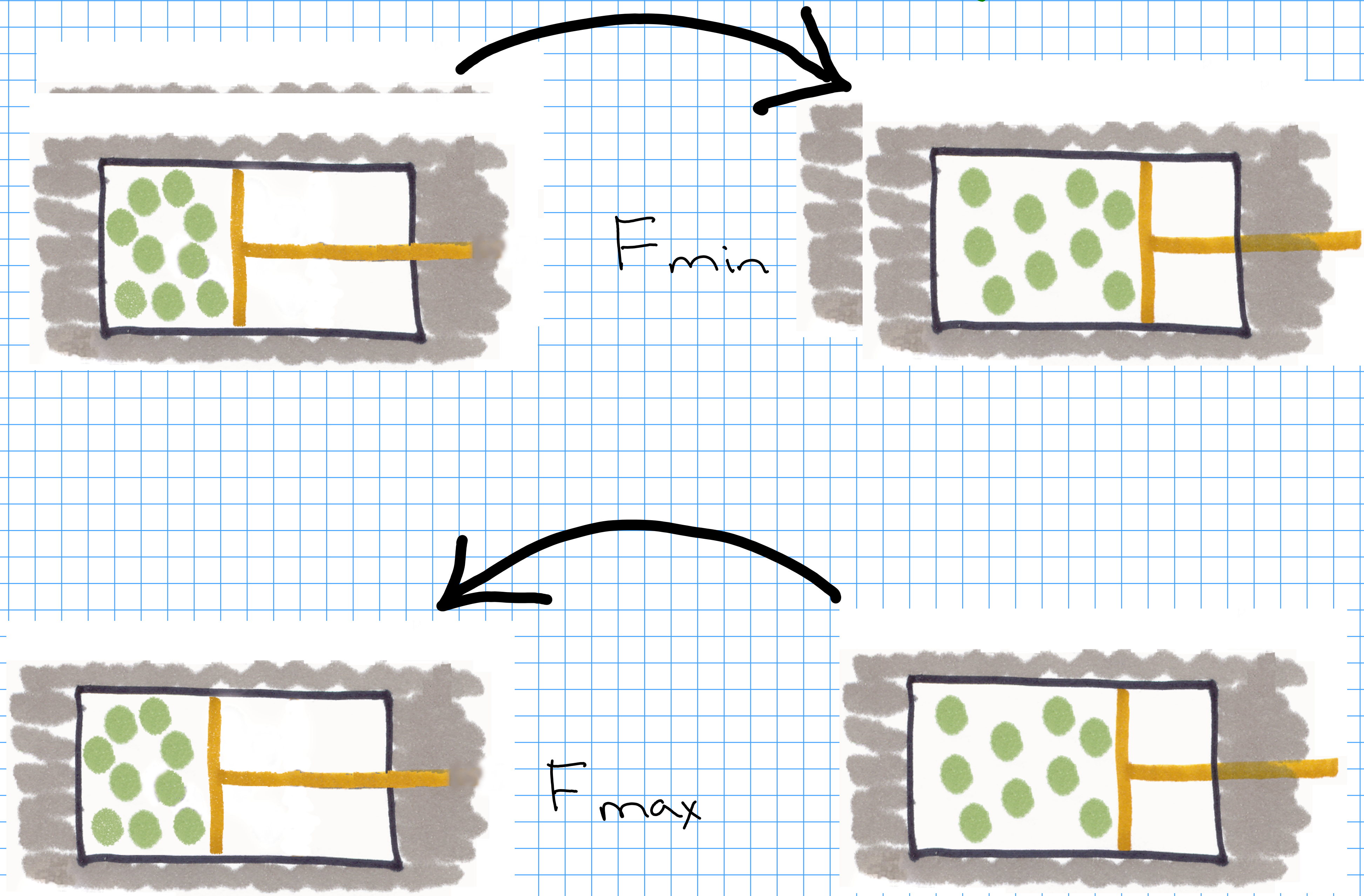
w.t

$\cancel{F = E - TS}$

F_{max}

F_{min}

Reversibility?



Not

Summary of Results

- Paradigm for Q.T.
- Distillation of work F^{min}
- Formation of states F^{max}
- Criteria for state-state transformations (thermo-majorization)
- irreversibility due to finite size effects
- irreversibility due to quantum coherences
- criteria for reversibility
- micro carnot cycle

Thermodynamics as a Resource Theory

- Janzig (2003)
- Horodecki,[2] Oppenheim (2003)
- Brandao, Horodecki, Oppenheim, Renes, Spekkens (2011)
- Horodeki, Oppenheim (2011)

Thermodynamics as a resource theory

Class of Operations	Thermal	Energy conserving U Heat bath γ at temp T Work as $\|0\rangle \to \|W\rangle$
Useless Set	Heat bath γ at temp T	
Monotones	$F = R(\rho\|\gamma)$	F_{max} F_{min}

(BHORS, 2011) (HO, 2011)

Thermal Operations

- (ρ_S, H_S) $\left(\tau_R = \sum \frac{e^{-\beta E_R}}{Z} |E_R, g\rangle\langle E_R, g|, H_R\right)$

  ↖ resource ↖ heat bath

- adding τ (in Gibbs state)

- energy conserving unitaries

  ↓ 1 st law ↓ 2, 3rd law

  - $H(t)$ with internal clock
  - U s.t. $[U, H] = 0$
  - H_{int} s.t. $\mathrm{tr} H_{int} \rho_{in} = \mathrm{tr} H_{int} \rho_{out} = 0$

$$F^{\varepsilon}_{min} = \sup_{\substack{\Pi \in \omega \Pi \\ \mathrm{tr}\,\Pi\omega \leq 1-\varepsilon}} -KT \ln \sum_{g,E} e^{-\beta E} \; h(g,E)$$

$$h(g,E) = \begin{cases} 1 & \text{if } p(g,E,\omega_\varepsilon) > 0 \\ 0 & \text{if } p(g,E,\omega_\varepsilon) = 0 \end{cases}$$

$$\omega = \sum_E P_E \, \rho \, P_E$$

$$F_{max} = \inf_{\rho_\varepsilon} KT \ln \min \{ \lambda : \rho_\varepsilon \leq \lambda \tau \}$$

$$\tau = \sum_{E,g} e^{-\beta E} / Z \; |E,g\rangle\langle E,g|$$

$$\| \rho_\varepsilon - \rho \| \leq \varepsilon \qquad (\rho_\varepsilon \text{ normalised})$$

Information Distances

$$R(\rho\|\sigma) = -\operatorname{tr}\rho\log\rho - \operatorname{tr}\rho\log\sigma$$

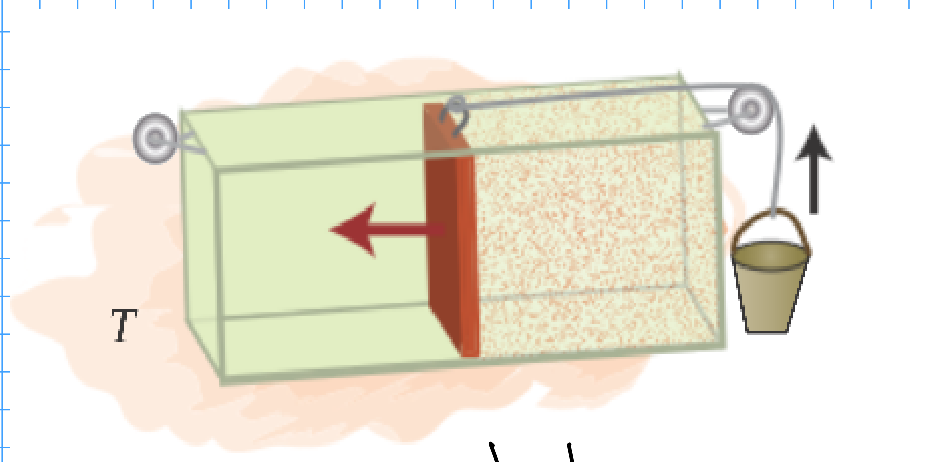
$$D^{\varepsilon}_{\min}(\rho\|\sigma) = -\ln\operatorname{tr}\Pi_\rho\sigma$$

$$D^{\varepsilon}_{\max}(\rho\|\sigma) = \inf_{\rho_\varepsilon} kT\ln\min\{\lambda : \rho_\varepsilon \le \lambda\tau\}$$

– Datta (2011)
Wang, Renner (2011)

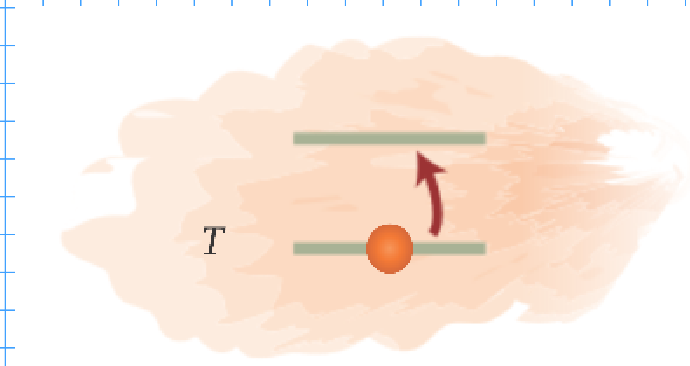
Work

Macro



weight

Micro



wit

$$\cancel{F = R(\rho\|\tau)}$$

$$\to D_\varepsilon^{max}(\rho\|\tau)$$

$$\to D_\varepsilon^{min}(\omega\|\tau)$$

c.f. Ahlberg (2011)

Work Distillation

exhaust

$$\rho_S \otimes \gamma_R \otimes |0\rangle\langle 0|_w \longrightarrow \sigma \otimes |1\rangle\langle 1|_w$$

resource — thermal bath — wit

$$H = H_R + H_S + H_W \qquad H_W = W|1\rangle\langle 1|$$

Energy conservation + Unitarity

$\rightarrow$ for each total energy block E

$$\text{rank}_{in} \leq \text{rank}_{out}$$

0^{th} order in $\delta E / \langle E \rangle$

$$F = \langle E \rangle - kT S(\rho)$$

$$\omega = \sum_E P_E \rho P_E$$

$$F_{min} \simeq \langle E \rangle - kT \ln \operatorname{rank} \omega$$

$$F_{max} \simeq \langle E \rangle + kT \ln p_{max}$$

$$\ln \operatorname{rank} \rho \geq S(\rho) \geq -\ln p_{max}$$

$$\operatorname{rank} \omega \geq \operatorname{rank} \rho$$

No work can be drawn from $|\omega\rangle = \sum_{E,g} \sqrt{\frac{e^{-\beta E}}{Z}} |E,g\rangle$

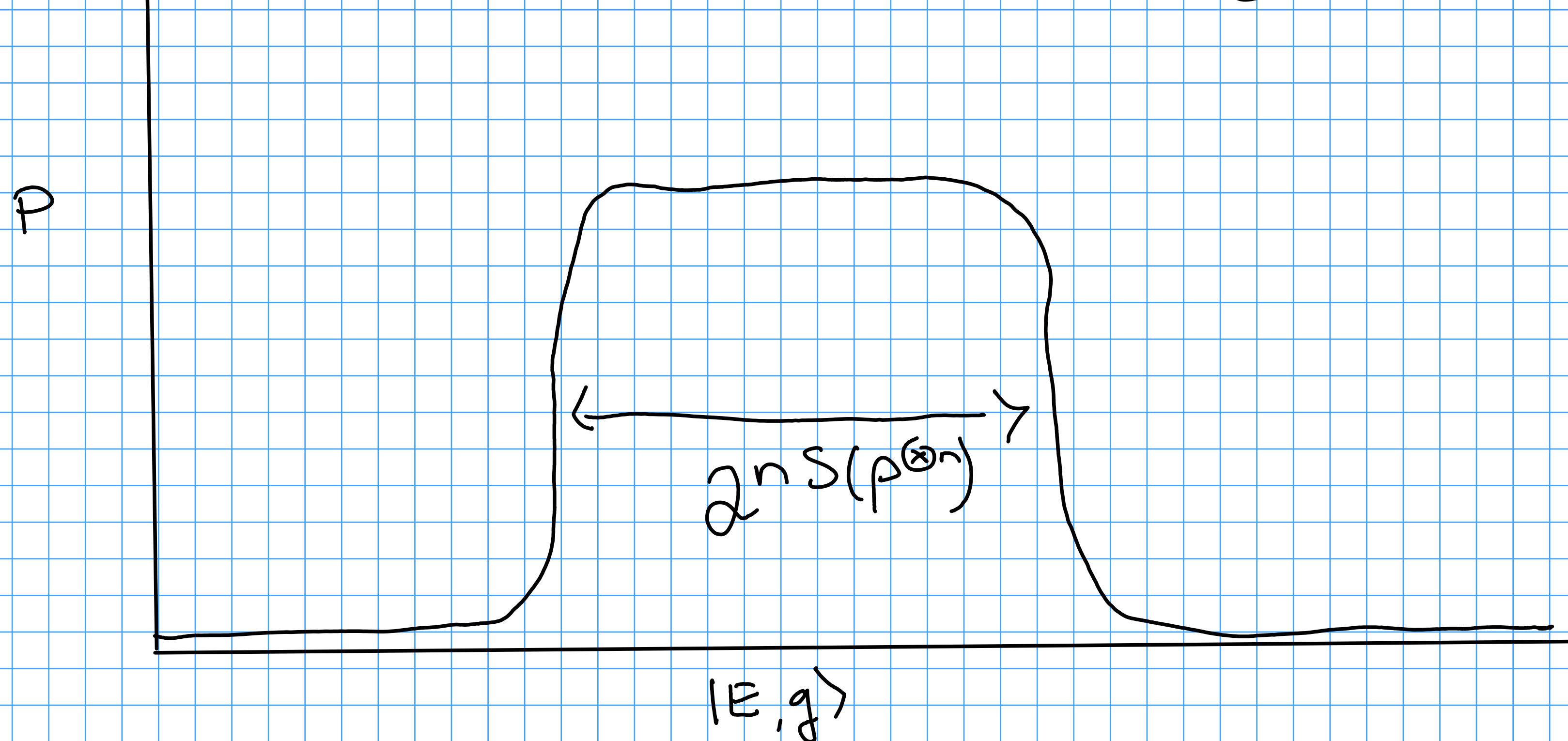
$$\boxed{F_{max} \gg F \gg F_{min}}$$

Allow probability ε of failure

(Dahlsten et. al. 2011)

Then sometimes $F \simeq F^{min}$

$$S(\rho^{\otimes n}) \simeq \ln \operatorname{rank} \omega_\varepsilon \quad \text{for } \|\omega_\varepsilon - \rho^{\otimes n}\| \leq \varepsilon$$



When are thermodynamical transitions possible?

$\rho \to \sigma$ iff ρ thermo-majorizes σ

recall majorization

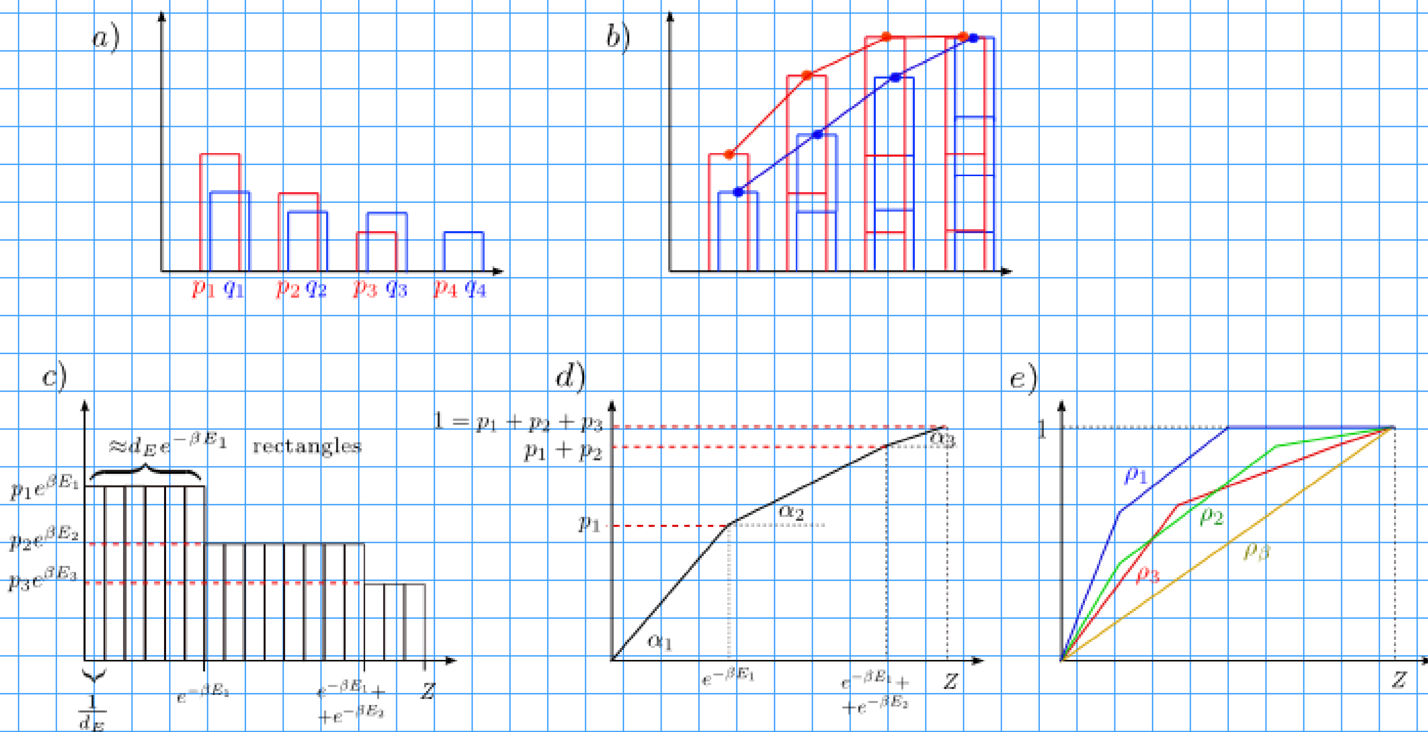
p_i of ρ in nonincreasing order

q_i of σ in nonincreasing order

i.e. $p_1 \geq p_2 \geq p_3 \dots$

$\rho > \sigma$ if $\sum_i^k p_i \geq \sum_i^k q_i \quad \forall\, k$

Thermo-majorization



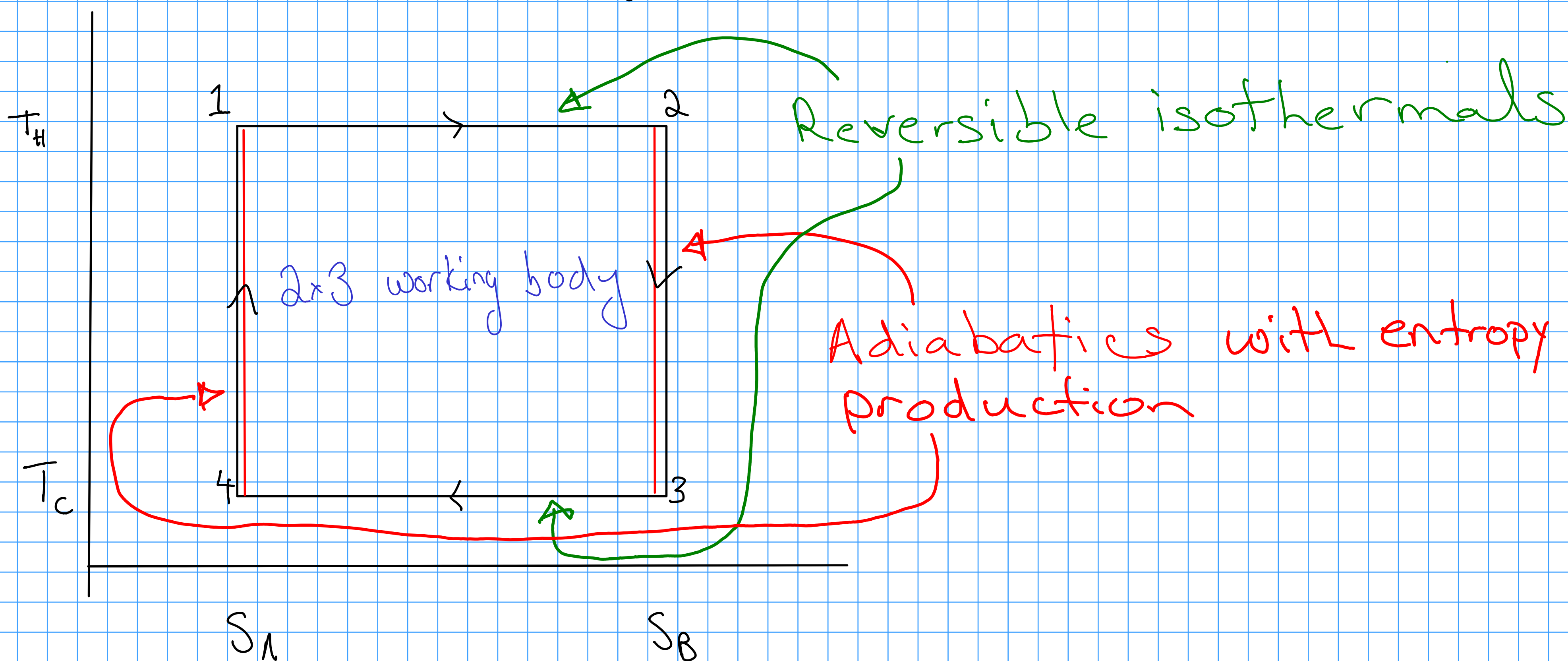
$$p(E_1, g_1)\, e^{\beta E_1} \geq p(E_2, g_2)\, e^{\beta E_2} \geq \ldots$$ β-ordered

Small heat engines

e.g. Lindon, Popescu, Skrzypczyk (2010) dim3, dim4

Are they efficient? No

Isothermal drawing is not reversible for a qubit



Conclusions

- Laws of thermodynamics
  - Uniteries
  - Energy Conservation
    → Heat bath preserving ops
- Two free energies → irreversibility
- Limitations due to finite size, quantumness
- Thermomajorization determines state trans
- Criteria for reversibility
- Small heat engines have prob to fail